A Synopsis of Recent Work in Edge Detection using the DWT

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Abstract

Automatic edge detection is a highly researched field because it is used in many different applications in image processing, such as diagnosis in medical imaging, topographical recognition and automated inspection of machine assemblies. Historically, the Discrete Wavelet Transform (DWT) has been a successful technique used in edge detection. The contributions of new, recent work in this area are examined and summarized concisely. Utilizing multiple phases, such as de-noising, preprocessing, thresholding coefficients, smoothing, and post-processing, are suggested for use with multiple iterations of the DWT in this research. The DWT is combined with various other methods for an optimal solution for the edge detection problem. This synopsis consolidates recent, related work into one source.

1. Introduction

Research in automatic edge detection is an active field because it is used in many different applications in image processing, such as diagnosis in medical imaging, topographical recognition and automated inspection of machine assemblies. Automatic edge detection within an image is a difficult task. When viewing an image, humans can easily determine the boundaries within an image without conscious thought. However, no single edge detection algorithm, at present, has been discovered which will automatically successfully discover all edges for many diverse images.

Conventional edge detection mechanisms examine the image's pixels for abrupt changes compared to its neighbors. This is often done by noting the pixels where the first derivative of the intensity is larger in magnitude than some threshold, or finding places where the second derivative of the intensity has a zero crossing. However, this method is performed only in the direct space domain. An example is Canny's popular edge detection algorithm, first in presented 1986 [1]. The problem with this type of traditional edge detection approach is that a low threshold produces false edges, but a high threshold misses important edges.

The introduction of the Discrete Wavelet Transform (DWT) brought ground-breaking influence in the area of signal analysis. Mallat, a pioneer in the field, established the idea that the wavelet transform, performed in a multi-scale manner, is effective for analyzing the meaning of the content in images. The DWT decomposes the image into sub-images, details and an approximation. We label the resulting sub-images from an octave (a single iteration of the DWT) as LL (the approximation), LH (horizontal details), HL (vertical details), and HH (diagonal details), according to the filters used to generate the sub-image. For example, HL means that we used a high-pass filter along the rows, and a low-pass filter along the columns. Then the DWT can be applied again and again, producing four more sub-images (with each successive iteration) that are 1/4 the size of the original image. The image can be reconstructed by undoing the transform. The channel with the low-pass filter contains the slow changing aspects, and the rapidly changing parts are preserved in the high-pass filter's channel. Edges in an image appear as sudden changes, and are therefore separated by this process. For more information on wavelets, please see [2].

Many recent, different methods that use the DWT because of its edge detecting characteristics are included in this review. The trait that all of the methods reviewed have in common is that multiple phases, such as de-noising, preprocessing, thresholding coefficients, smoothing, and post-processing, are suggested for use with multiple iterations of the DWT. Therefore, the DWT is combined with various other methods for an optimal solution for the edge detection problem.

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In this paper, the next section will cover the background of edge detection, section 3 will cover wavelets, and then section 4 will talk about the methods reviewed. Section 5 concludes the paper.

2. Edge Detection

Edge functions look for rapid changes in intensity. When the first derivative of the intensity is larger in magnitude than some threshold, or the second derivative of the intensity has a zero crossing, this often suggests an edge is present, because an edge is a curve that follows a path of rapid change in image intensity [3]. Edges are often associated with the boundaries of objects in a scene [4].

Traditional edge-detection algorithms belong to one of the following classes: gradient edge detectors, Laplacian of Gaussian (LoG), zero crossing, and Gaussian edge detectors. The gradient method looks for the maximum and minimum in the first derivative of the image. This classification includes algorithms such as Sobel, Prewitt, and Roberts. The Laplacian of Gaussian combines Gaussian filtering with a search for zero crossings in the second derivative of the image and was proposed by Marr and Hildreth [5]. Like LoG, the zero crossing method searches for the zero crossing in the second derivative of the image [6]. Finally, Gaussian edge detectors are symmetric along the edge and reduce the noise by smoothing the image. One of the most significant detectors here is Canny [1, 7].

Two basic properties of image intensity values are the basis for current segmentation algorithms; similarity and discontinuity [8]. The principal approaches in the similarity category are based on partitioning an image into regions that are similar to a set of predefined criteria. The approach for discontinuity algorithms is to partition an image based on abrupt changes in intensity. For this category, there are three basic types of intensity discontinuities in a digital image: points, lines and edges [8]. The low-pass and high-pass filters of the wavelet transform naturally breaks a signal into similar (low-pass) and discontinuous/rapidly-changing (high-pass) sub-signals, utilizing both basic properties. It is this characteristic that supremely suits the DWT for edge detection.

3. Wavelets

The Discrete Wavelet Transform (DWT) is currently used in a wide variety of signal processing applications, such as in audio and video compression, removal of noise in audio, and the simulation of wireless antenna distribution. Wavelets have their energy concentrated in time and are well suited for the analysis of transient, time-varying signals. Since most of the real-life signals encountered are time varying in nature, the Wavelet Transform suits many applications very well [9].

The 2-D discrete wavelet transform (DWT) decomposes the image into sub-images, 3 details and 1 approximation. The approximation looks just like the original; only on 1/4 the scale. The 2-D DWT is an application of the 1-D DWT in both the horizontal and the vertical directions. The DWT separates an image into a lower resolution approximation image (LL) as well as horizontal (HL), vertical (LH) and diagonal (HH) detail components. The low-pass and high-pass filters of the wavelet transform naturally break a signal into similar (low-pass) and discontinuous/rapidly-changing (high-pass) sub-signals. The slow changing aspects of a signal are preserved in the channel with the low-pass filter and the quickly changing parts are kept in the high-pass filter's channel. The fact that the DWT is a multi-scale analysis can be used to the edge detection algorithm's benefit. Multi-resolution is the process of taking one octave's LL output and putting this sub-image through another set of analysis filters. The first approximation will be used as a "seed" image and recursively apply the DWT a second and third time (or however many times it is necessary to perform to find all of the areas of interest) [9]. See [10] for more background on wavelets, and [11] for wavelet history.

4. Recent DWT Approaches

Various approaches using the DWT for edge detection will be discussed. Some of the methods are tailored for very specific applications [12], [13] and [14] while others offer generic edge detection solutions [15], [16], [17] , [18], and [19].

One approach for edge detection using the DWT, proposed by [15] uses two distinct families of wavelets. Two dissimilar wavelets are referenced because isotropic wavelets excel at isolating point wise discontinuities and directional wavelets shine at locating contoured paths; each type corrects a weakness in the other type of wavelet [15].

First, an image is decomposed using an ordinary, isotropic and separable wavelet transform. For this category of wavelet, the geometry of the wavelet is the same regardless of direction (rotational invariance) and is strongly vertical and horizontal in nature. Two examples of an isotropic wavelet are Gaussian or Mexican Hat. A matrix $W_j(x,y)$, containing the magnitude, for each pixel x and y, in each octave j, is produced, using only the vertical (v) and horizontal sub bands (h).

$$W_j(x,y) = \sqrt{|W_j^h(x,y)|^2 + |W_j^v(x,y)|^2} \quad [15]$$

Otsu's double thresholding method is then applied to $W_j(x,y)$ to obtain three classes: points that are not part of the edge, points which are part of the edge, and points that are candidate edges. Otsu's thresholding method is histogram-based and chooses a threshold that maximizes the between class variance [3] [20].

The points which are candidate edges are now examined to determine if they are part of the edge or not. This is accomplished by computing the phase angle θ_j, at each octave j, for each pixel that is in the class of the candidate edges.

$$\theta_j(x,y) = arctan(W_j^h(x,y)/W_j^v(x,y))$$

(x,y) is a candidate edge [15]

For pixels in which $\theta_j(x,y) = \theta_{j-1}(x,y)$, a directional wavelet transform will be performed. A directional wavelet is defined as a wavelet whose mother wavelet has support in a convex cone in the spatial frequency space with the apex at the origin. Examples of a directional wavelet are Gabor (Morlet) and Cauchy. Otsu's single thresholding method is then used to determine if each pixel is part of the edge or not [20]. As a result, every pixel is now classified as an edge, or not an edge.

The measurement used to score the results is called the *Figure of Merit F*, from Pratt. Four contrasting edge detection methods were scored: using only an isotropic wavelet with Otsu's lower bound threshold, using only an isotropic wavelet with Otsu's upper bound threshold, using the isotopic and directional wavelet method with the Gabor wavelet, and using the isotopic and directional wavelet method with the Cauchy wavelet. The best results were obtained with the Cauchy wavelet, followed by the Gabor wavelet [15].

In noiseless images with high contrast, Canny's edge detection has proven to be very successful [1]. However, noisy images are a different story, and results are not as good. To provide better edge detection for noisy images, and detect diagonal edge information, Lu and Zhang [16] use the wavelet transform with shifted coefficients.

The DWT (using the Haar wavelet for j = 1, 2, 3, 4 octaves) is performed on the image, finding the wavelet coefficients $w_j(x,y)$. Three shift operations are performed, one in the horizontal details, one in the vertical details, and one in the diagonal details, for the coefficients obtained in octave 3 and octave 4.

$$w_{4H} = max(w_4 \mathrm{X} w_{3L}, w_4, w_4 \mathrm{X} w_{3R})$$
$$w_{4V} = max(w_4 \mathrm{X} w_{3U}, w_4, w_4 \mathrm{X} w_{3D})$$
$$w_{4D} = max(w_4 \mathrm{X} w_{3DU}, w_4, w_4 \mathrm{X} w_{3DD})$$

w_{3L} is the coefficient from the 3^{rd} octave horizontal details, shifted left one pixel, w_{3R} is the coefficient from the 3^{rd} octave horizontal details, shifted right one pixel, w_{3U} is the coefficient from the 3^{rd} octave vertical details, shifted up one pixel, w_{3D} is the coefficient from the 3^{rd} octave vertical details, shifted down one pixel, w_{3DU} is the coefficient from the 3^{rd} octave diagonal details, shifted diagonally up one pixel, w_{3DD} is the coefficient from the 3^{rd} octave diagonal details, shifted diagonally down one pixel.

Now apply the shift again, substituting the w_3 and w_2 coefficients to obtain w_{3H}. Then shift again using the coefficients from w_2 and w_1, giving w_{2H}. w_{1H} equals w_1. Add $w_{1H} + w_{2H} + w_{3H} + w_{4H}$ to obtain w_{CH}. Obtain w_{CV}, and w_{CD} in the same manner. Threshold w_{CH}, w_{CV}, and w_{CD} to obtain binary images w_H, w_V, and w_D, respectively. w_{1H} convolved with w_H produces m_H. m_V and m_D will be formed in an analogous manner. Reconstruct $m_H m_V$, and m_D, using the inverse DWT to obtain the edge image.

The results appear to be very good visually for the one image tested (the bacteria.jpg [3]), compared to traditional edge detection methods such as Canny, Sobel, Prewitt, LoG, and Roberts. Only visual results are provided, and no quantitative scoring routine is discussed.

The work of Jung and Scharcanski [17], which is largely based on the work performed in [18] proposes combining the watershed method with the DWT. Watershed segmentation splits an image into regions using a geographical analogy for gray-scale images. Visualizing a gray-scale image as a topographical relief map in 3-D creates elevations and depressions. If water were to be poured on this surface, minima would collect in certain areas (the local minima) and run off of others (the local maxima). The objective of the watershed transform process is to convert an original image into a second image in which the features that are of significance become the basins.

The first step is pre-processing, which uses the DWT for de-noising, calculates the edge magnitudes from the coefficients obtained from the DWT, and then applies a special "shrinkage factor" on the DWT coefficients. These shrinkage factors are combined over multiple octaves using the harmonic mean. A de-noised image is obtained when the inverse wavelet transform is performed on the updated "shrunk" DWT coefficients. In the second step, the DWT is recalculated using the edge-enhanced image and the watersheds are computed on the magnitudes re-obtained from the new enhanced image. The last and final post-processing step involves merging smaller watershed regions that may be smaller than desired.

Three images were used to gather qualitative and quantitative results and compared against the competing methods of: segmentation using Gaussian filtering and the Prewitt operator, segmentation using morphological filtering, and Edge Flow. The quantitative results were reported as: CT scan - 14.57, 75.06, 75.84 and 21.55% (Proposed, watershed with Prewitt, morphological, and Edge Flow) and Industrial Image - 20.47, 27.50, 55.42 and 64.87% (Proposed, watershed with Prewitt, morphological, and Edge Flow).

The next topics discussed use the DWT for edge detection for specific applications. One of the applications for edge detection is to extract moving objects from video frames. For example, this technique could be helpful in automating video monitoring systems that try to locate intruders.

In the scheme proposed by [12], first, the wavelet domain change detection map W_d is found by performing the DWT on two frames, a current frame n, and previous frame $n-1$. If the difference in the coefficients for each pixel of the current frame and the previous frame is greater than a threshold th, the pixel $W_d(x,y)$ is set to 1, otherwise the pixel value is 0. Canny edge detection is run on W_d, producing matrix E_d [1].

Locate each of the current frame's (n) edges by using Canny again, giving E_n for a t number of frames. Find a background index (S_t) for each pixel by adding 1 to S_t if it is an active pixel ($E_n(x,y) - E_{n-1}(x,y) \neq 0$) . The background edges ($B_n(x,y)$) are equal to $E_n(x,y)$ if $S_t(x,y)$ is greater some threshold.

The initial matrix of moving edges M_0, can be calculated by subtracting the background edges, B_0 from the edge matrix E_0. Subsequent M_n values can now be computed.

$M_n = M_n^c \cup M_n^s$ [12]
M_n^c is the changed moving edge map
$M_n^c = e \in E_n | min||e-x|| \leq T_c$ [12]
$x \in E_d, T_c$ is a distance threshold
M_n^s is the still moving edge map
$M_n^s = e \in E_n | e \notin B_n, min||e-x|| \leq T_s$ [12]
$x \in M_{n-1}$
T_s is a distance threshold

Post-processing is applied to M_n to get rid of noise. The post-processing used is not stated.

To extract the Video Object Plane (VOP) information from M_n, the horizontal mask (the region inside edges row wise) and the vertical mask (the region inside edges column wise) is located. Then the morphological operation of erosion is used to fill in the image.

Scored results were obtained using a simple pixel-based quality measurement. Three varied videos were chosen, and the method in [12] outperformed (slightly) two other previously proposed methods, Kim's method [12] and HOS change detection [12].

With the rise in automated toll passes on the highways, and the rise in the number of people willing to run these tolls without paying, it has become attractive to have the capability to automatically detect license plates from a gray-level image. Hsieh, Juan and Hung present an extremely specialized idea to do just that [13].

The Haar wavelet is used to decompose the initial image. The resulting detail bands are converted to binary using a predefined threshold. In the LH (horizontal details) band find the reference line, which is the maxima peak on the horizontal projection. Ideally, this will be the top of the license plate.

The HL (vertical details) band will aid in calculating the license plate width. Two of the maximal peaks on the projection of the HL band will be the sides of the license plate. The two that are the furthest apart will be used as the default for the license plate width.

The region below the reference line and in between the vertical lines will be the candidate region for the license plate. To verify that this region is promising, some physical features of a license plate are checked against the candidate region. For example, license plates usually have a 3:1 ratio of length to width.

If the candidate region has been verified successfully, a column search and row search method is employed within the region to tighten up the region and discard extra outside edges. Using addition, the total of the pixel values in two contiguous columns is found. If the total of the two columns is below a threshold, the outer column may be discarded. This action is performed iteratively. The same routine is employed for rows.

Of 315 images used, accurate license plate region identification was obtained from 291 images. Images without much contrast between the license plate and the vehicle, or images in which the automobile was at a great distance failed [13].

Another algorithm is tailored specifically to locate breast carcinoma that have the characteristic of a solid center inside the mass region, and is presented by [14]. Initially, segmentation is performed to isolate the breast region from the background. The foreground is examined and iterative thresholding is used to extract any suspicious areas. The threshold t_0 is obtained by first finding the mean of the image, M. M_a is the mean of all of the pixels above t, and M_b is the mean of all of the pixels below t. $(M_a + M_b)/t$ will produce the next t. This operation is performed iteratively until t converges. The image containing only the breast area is convolved with the mean threshold template produces an image with the suspicious area highlighted.

The next step is to locate the masses within the area isolated in the last step. This is where the DWT is performed because a mass with a solid center will have a frequency close to zero in the high frequency bands in the second and third octaves. These locations will be registered.

The three octaves are then reconstructed, minus the LL (approximation) bands. The reconstructed image will contain some “black holes” and those will be registered. To erase tiny objects, the image is smoothed with a Gaussian filter. Then the regions where masses with a solid center lie are extracted by performing morphological erosion followed by dilation. This allows the centers of masses to be located. Canny edge detection is then carried out to find the edges of the suspect tumors.

Using 60 mammograms that contained 107 masses in 30 different patients, the results of the method proposed by [14] was compared to the diagnosis of radiologists. The results were correct in 93.6 % of the cases.

Threshold selection is extremely critical for proper edge detection. An adaptive method for thresholding is presented. This method responds to the statistics that are gathered for the characteristics of a particular image. This is necessary because images may be very different in many ways such as: their intensity levels, amount of contrast, amount of noise, and complexity.

The last method finds an adaptive threshold using wavelets [19]. First, perform the DWT using Symlet's 8 coefficient wavelet for 3 levels giving the wavelet coefficients, $w_{i,j}$. Find $\sigma_n = median(|w_{i,j}|)/0.6745$, $w_{ij} \in$ sub band HH (the approximation) in octave 1. σ_n is the estimation for the standard deviation of the noise. In all sub bands containing high frequency information, using σ_n, find $\sigma_{s,t}$, which is the noise-free signal variance:

$$\sigma_{s,t}^2 = \max\left(0, \frac{1}{n^2}\sum_{i,j=1}^{n} w_{i,j}^2(s,t) - \sigma_n^2\right)$$

where s is the scale, $t \in$ HH, HL, LH, $n \times n$ is the size of the sub-band. There will be 3 $\sigma_{s,t}$, for each scale and 3 levels, so there will be 9 σ_s, t, in total.

The adaptive threshold, $T_{s,t}$ can now be calculated:

$$T_{s,t} = \frac{2^{-(\sigma_{s,t})/(\sigma_n)\times s}}{\sum_{s,t} 2^{-s}} \times T_U$$

T_U is the D.J universal threshold (a threshold proposed by Donoho and Johnstone which gives lower weights to the higher scales and higher weights to the lower scales) defined by:

$$T_U = \sigma_n\sqrt{2\log M}$$

Use the $T_{s,t}$ calculated at each of the sub bands to threshold the image. Therefore, this threshold will be applied to the 9 high frequency images. Perform the inverse DWT on the 3 octave levels to obtain a noiseless image.

Selesnick, et al., discuss complex wavelet transforms (not to be confused with the continuous wavelet transform (CWT)). Wavelet analysis for certain applications can be enhanced by using a real transform of two branches, a real and an imaginary branch. That is, one branch is considered to be imaginary, though it uses a real wavelet and scaling filter pair.

Selesnick, et al., claim that image processing, specifically edge detection, can be enhanced with their technique. Interestingly, they propose using different filters at each stage of the transform. For example, in image processing, they claim that the transform is directionally selective, meaning that it does not spread out the edges the way that a separable 2-D DWT does.

Though the discrete wavelet transform has many benefits, such as efficient representation of the signal, Selesnick, et al., point out four issues with it: oscillations, shift variance, aliasing, and lack of directionality [21]. The wavelet functions oscillate in a wave-like pattern (thus the name). Thus, an edge in the signal generates both positive and negative values in the wavelet domain, so the edge does not stand out. Shift variance is a related issue. Suppose that we represent an impulse function in the wavelet domain; we might see large positive or negative values, depending on the location of the impulse. Aliasing is a third issue with the DWT. While the two channels (approximation and detail) in an octave recombine at the end to perfectly reproduce the signal seen at the input, the anti-aliasing is somewhat fragile. Finally, lack of directionality means that the multi-dimensional wavelet transform spreads out information, like ripples from a drop splashing in a pool of water. Selesnick, et al., propose the complex wavelet transform as a remedy to these four problems [21].

5. Conclusions

It is apparent that the DWT is a well accepted technique for edge detection currently, and is widely in use for solutions that require edge detection technology. All of the methods put forth are sophisticated and advantageous. However, if forced to locate a shortcoming, some scoring method or methods used to obtain the results that “perform better than using conventional filtering and edge detection techniques” [17] were not fully explained. Also, quite often a very small sample was used for the results. For example, Lu and Zhang used only one image and Chen and Han used only two images. As well, only one kind of noise, white noise, was tested [19]. This is not to suggest that the methods presented are not valid and useful, but that there are still areas remaining that can be refined, advanced and further tested.

References

[1] J. Canny, "A computational approach to edge detection," *IEEE Trans. Pattern Anal. and Machine Intell.*, vol. 8, pp. 679-698, 1986.

[2] S.Mallat, "Multifrequency channel decompositions of images and wavelet models," *The 7th International Conference on Advanced Communication Technology, ICACT 2005*, vol. 1, page(s): 115-120, Feb. 21-23 2005.

[3] Matlab Documentation, *Image Processing Toolbox User's Guide*, Release 7.1, The MathWorks, Inc. 3 Apple Hill Drive, Natick, MA 01760-2098, 2007.

[4] James R. Parker, *Algorithms for Image Processing and Computer Vision*, John Wiley and Sons, Inc., 1997 pages 1-28.

[5] D. Marr and E. Hildreth, "Theory of Edge Detection," *Proceedings of the Royal Society of London. Series B.*, Volume 207, 1980, pages 187-217.

[6] Stephane Mallat, "Zero-Crossings of a Wavelet Transform," *IEEE Transactions on Information Theory*, Volume 17, Number 4, 1991, pages 1019-1033.

[7] Mohsen Sharifi, Mahmoud Fathy, and Maryam Tayefeh Mahmoudi, "A Classified and Comparative Study of Edge Detection Algorithms", *Proceedings of the International Conference on Information Technology: Coding and Computing (ITCC.02)*, Las Vegas, Nevada, USA, April 8-10, 2002, pages 117-120.

[8] Rafeal C. Gonzalez, and Richard E. Woods, *Digital Image Processing*, Prentice Hall, 2007.

[9] Michael Weeks, *Digital Signal Processing Using MATLAB and Wavelets*, Infinity Science Press, 2006.

[10] Stephane Mallat, "A Theory for Multiresolution Signal Decomposition: The Wavelet Representation," *IEEE Pattern Analysis and Machine Intelligence*, Volume 11, Number 7, 1989, pages 674-693.

[11] Stephane Jaffard, Yves Meyer and Robert D. Ryan, *Wavelets Tools for Science and Technology*, Society for Industrial and Applied Mathematics (SIAM), 2001.

[12] Zhang Xiaoyan, Zhao Rongchun, Ma Zhiqiang, "A new method for moving object extraction automatically", *IEEE International Symposium on Communications and Information Technology, ISCIT 2005*, vol. 2, Oct. 2005 12-14, page(s): 1460-1463.

[13] Ching-Tang Hsieh, Yu-Shan Juan, Kuo-Ming Hung, "Multiple license plate detection for complex background", *19th International Conference on Advanced Information Networking and Applications, AINA 2005*, vol. 2, March 28-30 2005, page(s): 389-392.

[14] Weidong Xu, Shunren Xia, Min Xiao, Huilong Duan, "Model-based Algorithm for Mass Segmentation in Mammograms", *27th Annual International Conference of the Engineering in Medicine and Biology Society, IEEE-EMBS 2005*, Sept. 1-4 2005, page(s): 2543-2546.

[15] J. M. Niya, A. Aghagolzadeh , M. A. Tinati, and S. Feizi, "2-step wavelet-based edge detection using gabor and cauchy directional wavelets," *IEEE Trans. Acoustics, Speech, Signal Processing*, vol. 37, page(s): 2091-2110, December 1989.

[16] Qing-Hua Lu, Xian-Min Zhang, "Multiresolution edge detection in noisy images using wavelet transform", *Proceedings of 2005 International Conference on Machine Learning and Cybernetics*, vol. 8, August 2005 18-21, page(s): 5235-5240.

[17] Claudio Rosito Jung and Jacob Scharcanski, "Robust watershed segmentation using wavelets", *Image and Vision Computing*, volume 23, issue 7, July 2005, page(s):661-669.

[18] Claudio Rosito Jung and Jacob Scharcanski, "Robust watershed segmentation using the wavelet transform", *Proceedings. XV Brazilian Symposium on Computer Graphics and Image Processing*, Oct. 7-10 2002, page(s): 131-137.

[19] Y Chen and C Han, "Adaptive wavelet threshold for image denoising", *Electronics Letters*, vol. 41, issue 10, May 12 2005, page(s): 586-587.

[20] N. Otsu, "A threshold selection method from gray level histogram", *IEEE Trans. Systems. Man and Cybernetics*, vol. 9, March 1979, page(s): 62-66.

[21] Ivan W. Selesnick, Richard G. Baraniuk, and Nick G. Kingsbury, "The Dual-Tree Complex Wavelet Transform," *IEEE Signal Processing Magazine*, November 2005, pages 123-151.