Enhancing K-means and Kernelized Fuzzy C-means Clustering with Cluster Center Initialization in Segmenting MRI brain images

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Abstract—**Clustering is the process of organizing data objects into a set of disjoint classes called clusters. The objective of this paper is to develop an enhanced k-means and kernelized fuzzy c-means for a segmentation of brain magnetic resonance images. Performance of iterative clustering algorithms which converges to numerous local minima depend highly on initial cluster centers. In general the clustering algorithm chooses the initial centers in random manner. In this paper we propose a new center initialization algorithm for measuring the initial centers of the proposed clustering algorithms. This algorithm is based on maximum measure of the distance function which is found for cluster center detection process. More recently clustering is an effective tool in segmenting medical images for further treatment plan. In order to solve the problems of clustering performance affected by initial centers of clusters, this paper introduces a specialised center initialization method for executing the proposed algorithms in segmenting medical images. Experiments are performed with real brain images to access the performance of the proposed methods. Further the validity of clustering results are obtained using silhouette method and compares the results with the results of original k-means and fuzzy c-means clustering algorithms. The experimental results show the superiority of the proposed clustering results.**

Keywords- Clustering; K-means clustering; Fuzzy c-means clustering; Initial cluster centers; Image segmentation

I. INTRODUCTION

Clustering aims to analyze and organize data into groups based on their similarity. Clustering is an example of unsupervised classification. Classification refers to a procedure that assigns data objects to a set of classes. Unsupervised means that clustering does not depends on predefined classes and training examples while classifying the data objects. Cluster analysis seeks to partition a given data set into groups based on specified features so that the data points within a group are more similar to each other than the points in different groups [1]. Therefore, a cluster is a collection of objects that are similar among themselves and dissimilar to the objects belonging to other clusters. Clustering is an crucial area of research, which finds applications in many fields including bioinformatics, pattern recognition, image processing, marketing, data mining, economics, etc.

The k-means and fuzzy c-means algorithms start by initializing the cluster centers. The input vectors (data points) are then allocated (assigned) to one of the existing clusters according to the square of the Euclidean distance from the clusters, choosing the closest. The mean (centroid) of each cluster is then computed so as to update the cluster center. This update occurs as a result of the change in the membership of each cluster. The processes of re-assigning the input vectors and the update of the cluster centers is repeated until no more change in the value of any of the cluster centers.

Recently, fuzzy c-means of unsupervised clustering techniques used on established outstanding results in automated segmenting medical images in a robust manner [1,2,5,16]. Fuzzy c-means clustering [4,14,18] is successfully applied in many real world problems such as astronomy, geology, medical imaging, target recognition, and image segmentation. Among them, fuzzy c-means segmentation method has considerable benefits, because they could retain much more information from the original image than hard segmentation methods [4].

Two simple approaches to cluster center initialization are either to select the initial values randomly, or to choose the first k samples of the data points. As an alternative, different sets of initial values are chosen (out of the data points) and the set, which is closest to optimal, is chosen. However, testing different initial sets is considered impracticable criteria, especially for large number of clusters [5]. Therefore, different methods have been proposed in literature [1,12,15,17]. Clustering is an effective tool in segmenting the medical images for further treatment plan. Brain tissue is a complex structure and hence proper diagnosis of many brain disorders greatly depends upon accurate segmentation of the three brain tissues namely, white matter (WM), grey matter (GM) and cerebrospinal fluid (CSF) in brain MR image.

The rest of this paper is organised as follows. In section 2, basic concepts of the k-means algorithm and a short analysis of the existing clustering methods are briefly introduced. In section 3, basic concepts of the fuzzy c-means are briefly introduced. In section 4, center initialization algorithm is presented for initialising the initial cluster center for the proposed k-means and fuzzy c-means algorithms. Section 5 experimentally demonstrates the performance of the proposed method. Finally, section 6 describes the conclusion of this paper.

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II. K-MEANS CLUSTERING TECHNIQUE

Efficiency of the original k-means algorithm heavily relies on the initial centroids [3,5,6,7]. Initial centroids also have an influence on the number of iterations required while running the original k-means algorithm. The computational complexity of the original k-means algorithm is very high, specifically for massive data sets [3]. This paper presents an enhanced method for finding the better initial centroids and to provide an efficient way of assigning the data points to suitable clusters.

K-means procedure

The k-means clustering algorithm includes the following steps:

1. Selecting the number of clusters k with initial cluster centroids v_i, i = 1, ... 2, k.
2. Partitioning the input data points into k clusters by assigning each data point x_j to the closest cluster centroid v_i using the selected distance measure, for example, Euclidean distance, defined as

$$d_{ij} = \| x_j - v_i \|$$

where $X = \{x_1, x_2, ...x_n\}$ is the input data set.

3. Computing a cluster assignment matrix U representing the partition of the data points with the binary membership value of the j^{th} data point to the i^{th} cluster such that $U = [u_{ij}]$, where

$u_{ij} \in \{0,1\}$ for all i,j, $\sum_{i=1}^{k} u_{ij} = 1$ for all j and $0 < \sum_{j=1}^{n} u_{ij} < n$ for all i.

4. Re-computing the centroids using the membership values by

$$v_i = \frac{\sum_{j=1}^{n} u_{ij} x_j}{\sum_{j=1}^{n} u_{ij}} \quad \text{for all i .}$$

5. If cluster centroids or the assignment matrix does not change from the previous iteration, stop; otherwise go to step 2.

The k-means clustering method optimizes the sum-of-squared-error-based objective function Jw (U, v) then

$$J_w(U,v) = \sum_{i=1}^{k} \sum_{j=1}^{n} \|x_j - v_i\|^2 .$$

It can be spotted from the above algorithm that the k-means clustering method is quite sensitive to the initial cluster assignment and the choice of the distance measure.

III. KERNELIZED FUZZY C-MEANS

The fuzzy c-means algorithm has been broadly used in various patterns and image processing studies [18]. According to fuzzy c-means algorithm, the clustering of a dataset can be performed by minimizing an objective function for a known number of clusters. Fuzzy c-means is based on minimization of the following objective function:

$$J_m = \sum_{i=1}^{N} \sum_{j=1}^{C} u_{ij}^m ||x_i - v_j||^2 \quad , 1 \le m < \infty$$

where m is any real number greater than 1, uij is the degree of membership of xi in the cluster j, xi is the ith of p-dimensional measured data, vj is the p-dimensional center of the jth cluster, and ||*|| is any norm expressing the similarity between any measured data and the center. Here, N represents the number of data and C represents the number of cluster centers. Fuzzy partitioning is carried out through an iterative optimization of the objective function shown above, with the update of membership uij and the cluster centres vj by

$$u_{ij} = \frac{1}{\sum_{k=1}^{C}\left(\frac{\|x_i - v_j\|}{\|x_i - v_k\|}\right)^{2/(m-1)}}, \quad v_j = \frac{\sum_{i=1}^{N} u_{ij}^m x_i}{\sum_{i=1}^{N} u_{ij}^m}, \quad j = 1,2, \ldots C$$

This iteration will stop when $max_{ij}\left\{\left|u_{ij}^{(t+1)} - u_{ij}^{(t)}\right|\right\} < \varepsilon$, where ε is a termination criterion between 0 and 1, whereas t is the iteration step. This procedure converges to a local minimum or a saddle point of J_m.

A. Mathematical representation

This section proposes enhanced fuzzy c-means based kernel function by incorporating normed kernel function and effective initialization method [10]. This section uses the following standard objective function of fuzzy c-means to construct the objective function of enhanced fuzzy c-means.

$$J(U,V) = \sum_{i=1}^{N} \sum_{j=1}^{C} u_{ij}^m \|x_i - v_j\|^2 \qquad (1)$$

Where

1. U is a fuzzy partition of X.

2. $v=(v_1, v_2, ..., v_c) \in R^p$ with $v_j=(v_{1j}, v_{2j}, ..., v_{pj})^T \in R^p$ is the cluster center of the j^{th} cluster , $1j \le C$;

3. $\|x_i - v_j\|$ is distance between the object x_i and center v_j and $\| . \|$ is any inner product on R^p .

4. m is the weighting exponent(also called the degree of fuzzyfier) $m \in (1,\infty)$.

B. Kernelized objective function

The Euclidean distance in Eq. (1) is replaced by the distance function $||\gamma(x_i) - \gamma(v_j)||^2$ where γ is a map on feature space, and kernelized objective function is constructed as

$$J(U,V) = \sum_{i=1}^{N} \sum_{j=1}^{C} u_{ij}^m ||\gamma(x_i) - \gamma(v_j)||^2 \qquad (2)$$

The distance function Eq. (2) can be expressed in terms of inner product space as

$$\|\gamma(x_i) - \gamma(v_j)\|^2 = \{\gamma(x_i),\gamma(x_i)\} + \{\gamma(v_j),\gamma(v_j)\} - 2\{\gamma(x_i),\gamma(v_j)\}$$

And from the property of kernel function that $\{\gamma(x_i),\gamma(v_j)\} = K(x_i, v_j)$, the distance function Eq. (2) can be rewritten as

$$\|\gamma(x_i) - \gamma(v_j)\|^2 = K(x_i, x_i) + K(v_j, v_j) - 2K(x_i, v_j) \qquad (3)$$

Where i=1, 2… N and j=1, 2… C.

Using Eq. (3) the objective function in Eq. (2) can be modified as

$$J(U,V) = 2\sum_{i=1}^{N}\sum_{j=1}^{C} u_{ij}^{m}\{K(x_i,x_i) + K(v_j,v_j) - 2K(x_i,v_j)\} \quad (4)$$

We express $K(x_i,v_j)$ as kernel function as

$$K(x_i,v_j) = -\frac{[\sum_{s=1}^{p} |x_{is} - v_{js}|^{\xi_1}]^{1/\xi_1}}{\xi_1} + \xi_2, \quad \xi_1, \xi_2 > 0 \quad (5)$$

Where ξ_1, ξ_2 are parameter which can be adjusted by users.

The effective robust fuzzy c-means based kernel function is given by

$$J(U,V) = 2\sum_{i=1}^{N}\sum_{j=1}^{C} u_{ij}^{m}\,(\xi_2 - K(x_i,v_j)) \quad (6)$$

The proposed partition matrix in an objective function satisfies the following conditions:

$0 \le u_{ij} \le 1$, for $1 \le i \le N$, $1 \le j \le C$,

$0 < \sum_{i=1}^{N} u_{ij} < N$, for $1 \le j \le C$,

$\sum_{j=1}^{C} u_{ij} = 1$, for $1 \le i \le N$. (7)

C. Evaluation of memberships partitioned

We are minimizing the objective function given in Eq. (6) with respect to uij , subject to the constraint (7) using Lagrangian multiplier method and consider the Lagrangian λ1,λ2,…..,λN , and we obtain the following for all i=1 and for all j

$$u_{11} = \left(\frac{\lambda_1}{2m\,(\xi_2 - K(x_1,v_1))}\right)^{1/(m-1)}$$

$$u_{1C} = \left(\frac{\lambda_1}{2m(\xi_2 - K(x_1,v_c))}\right)^{1/(m-1)}$$

Summing up all $u_{11}, u_{12} \ldots u_{1C}$.

Since $\sum_{j=1}^{C} u_{ij} = 1$, for $1 \le i \le N$, we have

$$\left(\frac{\lambda_1}{2m}\right)^{1/(m-1)} = \frac{1}{\left[\sum_{l=1}^{C}\left(\frac{1}{\xi_2 - K(x_1,v_l)}\right)^{1/(m-1)}\right]} \quad (8)$$

Using (8) we have

$$u_{11} = \frac{\left(\frac{1}{\xi_2 - K(x_1,v_1)}\right)^{1/(m-1)}}{\sum_{l=1}^{C}\left(\frac{1}{\xi_2 - K(x_1,v_l)}\right)^{1/(m-1)}} \quad (9)$$

In general we have

$$u_{ij} = \frac{\left(\frac{1}{\xi_2 - K(x_i,v_j)}\right)^{1/(m-1)}}{\sum_{l=1}^{C}\left(\frac{1}{\xi_2 - K(x_i,v_l)}\right)^{1/(m-1)}} \quad (10)$$

The general equation is used to obtain the membership grades for objects in data for finding meaningful groups.

D. Evaluation of successive centers or prototypes

Minimizing the objective function (6) with respect to v_j we obtain prototype updating equation as

$$v_c = \frac{\sum_{i=1}^{N} u_{iC}^{m} x_i}{\sum_{i=1}^{N} u_{iC}^{m}} \quad (11)$$

In general we have

$$v_j = \frac{\sum_{i=1}^{N} u_{ij}^{m} x_i}{\sum_{i=1}^{N} u_{ij}^{m}} \quad j=1,2,\ldots,C \quad (12)$$

E. Kernelized fuzzy C-means algorithm

The Kernelized fuzzy c-means algorithm includes the following steps

Step 1: Get the data from Image.

Step 2: Fix the number of Clusters and assign the initial cluster centers using center initialization algorithm.

Step 3: Compute partition matrix using (10).

Step 4: Update the cluster centers using (12).

Step 5: Repeat steps (3–4) until the following termination criterion is satisfied: $\|V^{(present)} - V^{(previous)}\| < \epsilon$ where $V^{(present)}$ and $V^{(previous)}$ are the vector of cluster prototypes at present iteration and previous iteration.

IV. Cluster Center Initialization

Mostly, the number of successive centers and results of k-means and fuzzy c-means clustering depend on initial centers of clusters [8]. In general the clustering algorithms choose the initial centers in random manner, which are affecting the results of clustering. Recently researchers have involved for choosing the initial centers with some meaningful algorithms [9,11]. In our work we propose a new center initialization algorithm for measuring the initial centers of proposed clustering algorithms. We measure the initial centers based on the following proposed center initialization algorithm.

A. Center initialization

Read the brain MRI medical image into matrix form. The image height measurement is denoted as M and the image width measurement is N. The image is now in the form of

$$X = \begin{bmatrix} X_{0,0} & X_{0,1} & \cdots & X_{0,N-1} \\ X_{1,0} & X_{1,1} & \cdots & X_{1,N-1} \\ \cdots & \cdots & \cdots & \cdots \\ \cdots & \cdots & \cdots & \cdots \\ X_{M-1,0} & X_{M-1,1} & \cdots & X_{M-1,N-1} \end{bmatrix}$$

Where X is image data.

The image data is next converted into linear data, and the data dimension must be (1,MxN). The mathematical representation of the data is

$X' = [X_0, X_1, X_2 \ldots\ldots X_{MxN-1}]$

Here X' is the linear data.

The data is now in the range of 0 to 255. These gray levels are spread in linear way. This data should be rearranged to produce a way for efficient clustering. After sorting data sets arrangements must be

X'' = sorted function(X'), also $X''_0 <= X''_1 <= X''_2 <= X''_3 \ldots\ldots <= X''_{MxN-2} <= X''_{MxN-1}$

Where X'' is ascending order of X'.

The total cluster number is decided by the user and X' should be partitioned in such a way that

$X' = \{ [X'_{0}, X'_{1}, X'_{2} \ldots\ldots\ldots\ldots X'_{1xg-1}] \; [X'_{1xg+0}, X'_{1xg+1}, X'_{1xg+2} \ldots\ldots\ldots\ldots X'_{2xg-1}][X'_{2xg+0}, X'_{2xg+1}, X'_{2xg+2} \ldots\ldots\ldots\ldots X'_{3xg-1}] \; [X'_{(c-1)xg+0}, X'_{(c-1)xg+1}, X'_{(c-1)xg+2} \ldots\ldots\ldots X'_{(c-1)xg-1}]\}; \quad g = \left\lfloor \frac{(MxN)}{C} \right\rfloor$

Here g is partition length and C is number of clusters. C should be as 1<C<(MxN).The term g represents how many elements should be grouped together for each partition. The sorted data X" is now portioned into C partition with the following level of elements set.

Elements count (P_0) = g

Elements count (P_1) = g

.

Elements count (P_{c-1}) = g + mod (MxN, C)

Here elements count function means the amount of element should lie in that partition. All partition have equal amount of elements except C-1 partition. Because the partial (remaining or unallotted) elements are assigned to C-1 partition in addition with the g amount of data.

The maximum measure of the distance function is found for the cluster center detection process. For this process each and every elements are subtracted with each other. In this distance values, the maximum measurement is determined and the two elements which generated that maximum measure is extracted and that corresponding data values are averaged to get the clustering center of the considered group. In this manner for all groups the clustering centers are generated. Consider Index 1 and Index 2 are maximum distance measure producing indexes.

$$\text{Clustering center} = \left\lfloor \frac{s(index1)+s(index2)}{2} \right\rfloor$$

B. Center initialization algorithm

- Read the image.
- Convert to linear form.
- Sort the data.
- Partition the data.
- Find maximum measure.
- Find cluster center.
- Repeat steps 5 to 6 until the cluster centers are found for all groups.

V. EXPERIMENTAL RESULTS

This section portrays some experimental results on real data on brain MRI. All the input dataset (total images are 80 for abnormal and normal) used for segmentation consists of T1 and T2 weighted, 256x256 pixel MR brain images. The popular clustering algorithms are k-means and fuzzy c-means in these methods the quality of the final clusters relies heavily on the initial centroids which are selected randomly. The proposed methods find the initial centroids and provide an efficient way of assigning the data points to the suitable clusters. The images considered as test images for applying k-means and kernelized fuzzy c-means algorithms are shown in Fig.1. They are original MRI T1 and T2 weighted normal (a-b) and abnormal (c-d) brain images. Fig. 2 (a-d) present clustering results of k-means method. Fig. 3 (a-d) present the results of proposed enhanced k-means. Fig. 4 (a-d) presents the results of segmented image by fuzzy c-means. Fig. 5 (a-d) presents the results segmented image by enhanced kernelized fuzzy c-means.

Clustering algorithms divide the images into four tissue classes in the brain: white matter, cerebrospinal fluid in the sulci, ventricles and a tumor. the number of clusters in the multi-dimensional feature space thus represents the number of classes in the image. Table 1 and table 2 give the segmentation accuracy of the two algorithms on brain images, where segmentation accuracy is defined using silhouette method in Ref. [13]. These silhouette average value measures the degree of confidence in the clustering assignment of a particular observation, with well clustered observations having values near 1 and poorly clustered observations having values near -1. The silhouette width s(i) of the object i is obtained using the equation

$$S(i) = \frac{(b(i)-a(i))}{max\{a(i),b(i)\}}$$

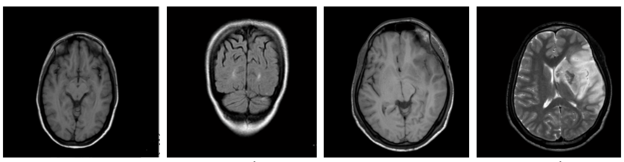
For each object we denote by the cluster to which it belongs and compute $a(i) = \frac{1}{|A|} - 1\sum_{j \in A, i \neq j} d(ij)$. Here a(i) is the average distance between the ith data and all other objects in the cluster A. And $b(i) = \min_{C \neq A} d(i,C)$. The cluster B which attains this minimum (that is d(i,C) = b(i)) is called the neighbour of object i, this is the second best cluster for object i.

TABLE I. SEGMENTATION ACCURACY FOR K-MEANS

Clustering methods	**Brain image**	
	Silhouette value	***Accuracy (%)***
K-means	0.36	36
Proposed EK-means	0.79	79

TABLE II. SEGMENTATION ACCURACY FOR FUZZY C-MEANS

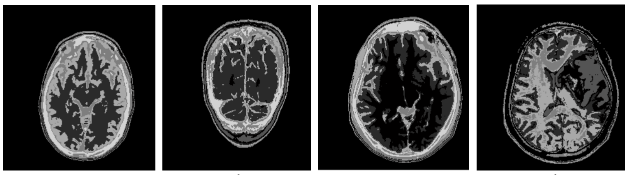
Clustering methods	**Brain image**	
	Silhouette value	***Accuracy (%)***
Fuzzy c-means	0.41	41
Proposed EK fuzzy c-means	0.83	83



a b c d

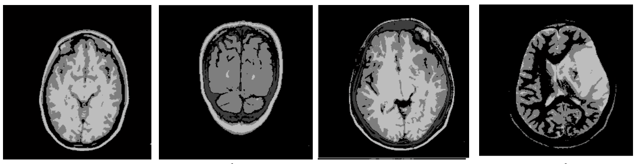
Figure 1. Original MRI T1 and T2 weighted normal (a,b) and abnormal (c,d) brain images

From Table 1 and table 2 proposed algorithms were obtained the best segmentation accuracy during the experiment on brain MRI. On the whole, proposed methods achieve better segmentation results under brain images.



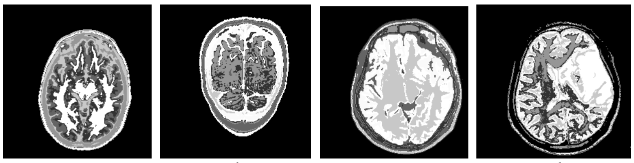
a b c d

Figure 2. Experimental results segmented image by K-means for normal (a,b) and abnormal (c,d) brain images.



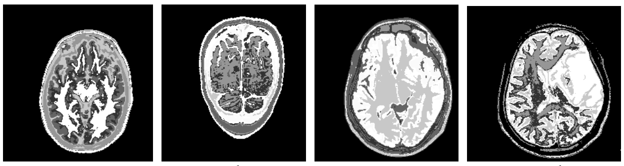
a b c d

Figure 3. Experimental results segmented image by proposed EK-Means for normal (a,b) and abnormal (b,c) brain images.



a b c d

Figure 4. Experimental results segmented image by fuzzy c-means for normal (a,b) and abnormal (c,d) brain images.



a b c d

Figure 5: Experimental results segmented image by enhanced kernelized fuzzy c-means for normal (a,b) and abnormal (c,d) brain images.

VI. CONCLUSION

In this paper we have presented an enhanced k-means and kernelised fuzzy c-means clustering with improved cluster center initialization algorithm to segment MR brain images. The proposed methods have higher segmentation accuracy in clustering complex dataset of brain medical images. The performance of k-means and kernelized fuzzy c-means algorithm depend on initial cluster center. The proposed methods have selected the initial center by using specialised method center initialization algorithm of this paper. By observing the result it can be said that k-means and kernelized fuzzy c-means can be successfully applied for clustering based segmentation of MRI images hence it is concluded that for MRI brain images, kernelized fuzzy c-means gives best clustering. For the experimental work, to evaluate the effectiveness of the proposed methods, real brain images were used for segmentation of different tissue classes. It is clear from our comparison that the proposed k-means and kernelized fuzzy c-means with center initialization algorithm performed well than k-means and fuzzy c-means with random initialization. The segmentation accuracy is obtained using shilhouette index value to demonstrate the superiority of the proposed approaches. The results reported in this paper shows that the proposed algorithms are an effective approach to construct a robust image segmentation algorithm.

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