

# Medical Image Restoration Using Optimization Techniques and Hybrid Filters

<sup>1</sup>XXXXXXXX, B.E., Information and Technology, Final Year, Department of Computer Science, Sathyabama University, Chennai – 15. Email-id: [xxxxxxxxxxx@gmail.com](mailto:xxxxxxxxxxx@gmail.com)

*Abstract— In clinical context, Medical images plays the most significant role. Medical imaging brings out internal structures concealed by the skin and bones, as well as to diagnose and treat diseases like cancer, diabetic retinopathy, fractures in bones, skin diseases etc. The medical imaging process is different for different type of diseases. The image capturing process contributes the noise in the medical image. Henceforth, captured images need to be noise free for proper diagnosis of the diseases. In this paper, we discourse various noises that affect the medical images and also accompanied by the denoising algorithms.*

Digital Image Processing technology executes computer algorithms to realize digital image processing which implies digital data modification that improves quality of the image. For medical image information extraction and foster analysis, the implemented image processing algorithm maximizes the clarity and sharpness of the image and also interesting features details. Initially, the computer is inputted with a digital image and programmed to process the digital image data provided with series of equations. The computer stores each pixel or picture element computation values. But, Noise in the digital image is a basic problem. Generally, Image noise is an unwanted spin-off while capturing image which blots out the wanted information. Many filters are available to remove noise, but when the noise is single type of noise such as salt and pepper, speckle noise or Gaussian noise etc. The filters suffer to remove the noise merely when the image is degenerated by mixed types of noise. To overwhelm this case, we introduce an optimization technique and Hybrid filter, where the Hybrid filter is composite of dissimilar filter which removes mixed type of noise from digital image.

**Key Terms:** Hybrid Denoising, Image Restoration, Optimization technique, Hybrid Genetic Algorithm, Hybrid Filters

## I. INTRODUCTION

Digital images play vital role in the day to day life of the human. It provides useful information like weather forecasting data using satellite cameras, traffic monitoring data and medical imaging data using X-rays, medical resonance imaging (MRI), Computed Tomography (CT), ultrasound imaging etc. Majority of images captured with the imaging instruments incur some surplus data or noise

which is not pertained to the main image. The incurred noise seems to appear in different form in images that is generally random information added or multiplied by the main image.

The noise incurred in the medical images increases difficulties during interpretation of images. Frequently Denoising process is to be done in earlier before analyzing the medical images, where the analyzing includes Segmenting, Classifying and Detecting diseases or injury. The multiplicative or additive noise found in the medical image are reduced by performing Denoising process. To reduce the noise, Medical image noise reduction technique introduces some approaches like Spatial domain filtering and Transform domain filtering. The Spatial domain filtering is further classified into linear and Adaptive Filtering. The Transform domain filtering is further classified into spatial-frequency filtering and wavelet filtering. The aforementioned methods have some limitations. For instance most of the filters are unable to maintain edge and textures of images as well as possible. Majority of filters use different quality evaluation metrics like RMSE (Root-Mean-Square Error), SNR (Signal-to-Noise Ratio) and PSNR (Peak Signal-to-Noise Ratio) for evaluating the performance of filters.

The remaining section of this paper is organized as follows. Section II depicts the basic idea related to the proposed work. The section III provides introduction to the proposed system and section IV furnishes the implementation details followed by result in section V. Finally, the section V concludes the paper.

## II. EXISTING SYSTEM AND RELATED WORK

**Existing System:** Some approaches are introduced earlier in order to reduce the noise of images, where the approaches include Spatial domain filtering and Transform domain filtering that are further classified into non-linear and linear filters and spatial-frequency filtering and wavelet filtering respectively. But, these methods induce some limitations. Namely most of the filters are unable to preserve edge and textures of image. Different quality evaluation metrics are utilized such as RMSE, SNR and PSNR for evaluating the filters performance efficiency and some additional assessments are available like visual assessment and texture analysis that could be inquired later.

The most common image denoising technique relies on filters, where the filters smooth the images in order to suppress noise. Nevertheless, the applied technique demeans

image important features such as edges, corners or texture. The filters suppressing the noise in the image are classified into linear filter and Adaptive Filter. The linear filter can be verbalized as convolution of a kernel (filter) through a noise image to produce the resulting image. On the other hand, Adaptive Filter is unable to perform convolution operation. The linear filter widely used for image denoising is the Wiener filter, which minimizes the mean square error among the recovered image and the original image. The most commonly used Adaptive Filter is the Guided Filter that replaces the value of each pixel by the median value of neighborhood pixels.

Some other techniques are available that highly aims in removing as much noise as possible and attempting to preserve image important features. The total variation (TV) denoising methods involves considering the noisy signals in an image, where the noisy signals have high total variation. The TV denoising method performs the denoising process by filtering these noisy signals. Anisotropic and isotropic diffusion processes use a function for identifying the edges in an image. The aforementioned techniques diffuse the image in a continuous manner for smoothing the image, but the problem is identifying when to stop the diffusion process through this edge-aware function. The resultant image is smoothed and its edges are preserved.

#### A. REVIEW OF LITERATURE:

Even though BM3D is counted as state-of-the-art in image denoising and also considered as substantial engineered method, Burger et al. depicted a plain multi layer perceptron (MLP) attains standardized denoising performance efficiency.

Recent addition in Image denoising literature are denoising auto encoders, which are utilized as a building block for deep networks introduced by Vincent et al. The introduction by Vincent et al is an extension to classic auto encoders. The denoising auto encoders can be stacked by forming a deep network that feeds the output of one denoising auto encoder to the other auto encoder below the above.

Jain et al. aimed proposing image denoising using convolution neural networks. This achieves performance better than state-of- the-art by observing a small sample of training images based on wavelets and Markov random fields. Xie et al. employed stacked sparse autoencoders to perform image denoising and inpainting processes, which realized at par with K-SVD. Agostenelli et al. tried out image denoising with adaptive multi column deep neural networks. The system is built using a combination of stacked sparse autoencoders to ensure robustness against different noise types.

### III. PROPOSED METHOD

**Proposed System:** We propose a method for medical image denoising using Hybrid Filters. The proposed method investigates image enhancement problem when the source image is debauched by Gaussian noise, Riccian noise, Impulse noise, and blurriness. The image edges corruption is assumption for the images found through scanning, transmitting, compression. To overcome such issues we propose an efficient and simple algorithm named Hybrid Filter based on the genetic algorithm for image Optimization, and Hybrid Filters like NL Filter, Hybrid Median Filter, and Guided Filter.

#### A. TYPE OF NOISES:

The noise affecting quality of image are detailed beneath. Classification of the noises are done according to their nature respectively.

**Gaussian Noise:** Gaussian noise is disseminated all over the image, the pixel value is represented by summation of the true pixel value and Gaussian distribution. The Gaussian noise is formed with a bell shaped structure and it is expressed as:

$$F(g) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(g-m)^2}{2\sigma^2}}$$

Where, g is the gray level of the pixel m is the mean value  $\sigma$  is the standard deviation of the noisy image

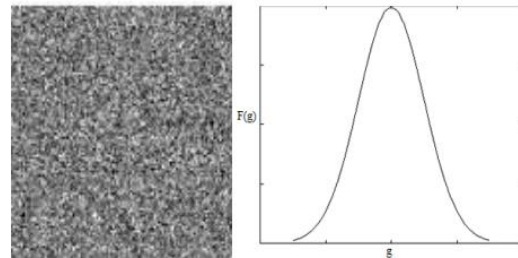


Figure (a) Gaussian noise image (b) Gaussian noise distribution

Fig. (a) depicts image with the Gaussian noise of zero mean and 0.05 variance and Fig (b) illustrates respective Gaussian distribution.

**Salt and Pepper Noise:** This noise is impulse-type called as intensity spikes noise. The noise is generated in the transmission channel. The noise is formed with small black (minimum intensity pixel value) and white (maximum intensity pixel value) dots hence it is called as salt and pepper noise. The value of salt and pepper noise in 8 bit grayscale image is 255 and 0 respectively. This noise is developed with camera sensors defects, improper pixel elements and faulty memory location. The salt and pepper noise appearance is shown in Fig.

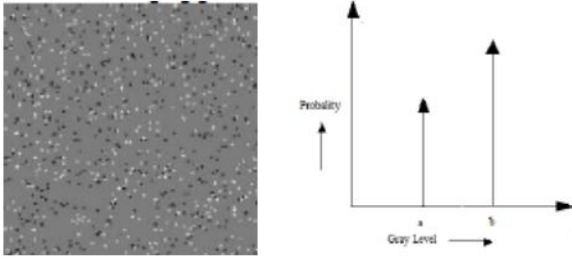


Fig: salt and pepper noise

**Speckle Noise:** Speckle noise happens in the SAR, LASER and acoustic images. The speckle noise is formed as multiplicative noise. The speckle noise occurs in almost all coherent imaging. It is given by

$$F(g) = \frac{g^{\alpha-1}}{(\alpha-1)!a^\alpha} e^{-\frac{g}{a}}$$

Where,  $a^2\alpha$  is the variance  
 $g$  is the gray level of the image

Fig. (a) Illustrates speckle noise image and Fig (b) gamma distribution plot.

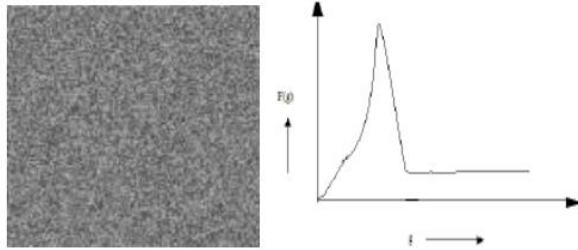
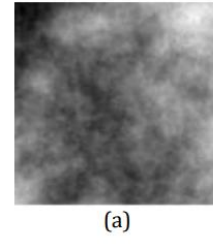
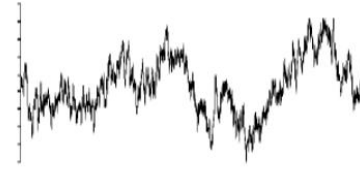


Fig: (a) Speckle noise (b) Gamma distribution

**Brownian Noise:** Brownian noise is fractal type of noise called as 1/f noise, and the mathematical model is given by the Brownian movement. The Brownian noise is generated by the non-stationary stochastic process. Graphical representation of the noise is shown in Fig.



(a)



(b)

Fig : (a) Brownian noise (b) Brownian noise distribution Image

#### IV. METHOD OF IMPLEMENTATION

##### A. HYBRID GENETIC ALGORITHM:

The Hybrid Genetic Algorithm (HGA) declared oneself in this work is based on the Genetic Algorithm (GA). The noisy image  $I$  is inputted and creation of the population by applying mutation operators on the noisy image. Moreover, some of the crossover operators are the same introduced and our work combines the GA that is different from the GA used earlier by using image denoising techniques. Evolutionary process is guided by the different fitness function. Pseudocode is generated by Algorithm 1, where the Pseudocode summarizes the Hybrid Genetic Algorithm functioning.

Hybrid Genetic Algorithm is executed for a fixed amount of time, where the same population develops while the best individual is not updated for  $\maxIter$ . The population is reset when this criterion is met and the best individual is held while the balance new population is formed by performing the same process repeatedly on the initial population.

Apiece individual is constituted by a two-dimensional array of pixels with integer values in the range  $[0, 255]$ . An initial population is created by different mutation operators over  $I$ . Hybrid Genetic Algorithm is directed by the objective function for performing best individual search. The objective function is expressed in Equation (1):

$$fitness(I) = \left( \sum_{\Omega} \sqrt{1 + \beta^2 |\nabla I|^2} \right) + \frac{\lambda}{2} (I - I_0)^2$$

which is an edge-aware feature-preserving diffusion flow function described earlier. The  $I$  parameter is the image undergoing evaluation,  $I_0$  is the noisy image,  $\beta$  and  $\lambda$  are balancing parameters and  $\Omega$  is the set of all points in the image.

---

**Algorithm 1** Proposed genetic algorithm.

---

```
1: procedure GENETIC RESTORATION II(Image Img)
2:   Pop  $\leftarrow$  createPopulation(Img);
3:   best  $\leftarrow$  Pop.best;
4:   while elapsedTime < maxTime do
5:     cont  $\leftarrow$  0;
6:     while cont < maxIter do
7:       IntermPop  $\leftarrow$  Pop
8:       for i  $\leftarrow$  1 to popSize do
9:         ind1, ind2  $\leftarrow$  fathers(Pop);
10:        ind3  $\leftarrow$  crossover(ind1, ind2);
11:        if mutation ? then
12:          mutate(ind3);
13:        end if
14:        IntermPop.append(ind3);
15:      end for
16:      sort(IntermPop);
17:      Pop  $\leftarrow$  IntermPop[1..popSize];
18:      if best = Pop.best then
19:        cont  $\leftarrow$  cont + 1;
20:      else
21:        cont  $\leftarrow$  0;
22:      end if
23:    end while
24:    reset(Pop);
25:  end while
26:  return Pop.best;
27: end procedure
```

---

and new individuals generated. Crossover operators create these new individuals, where the parents of new individuals are determined by tournament selection. When the parents are chosen, crossover operator selection process is performed to create a new individual. Three types of crossover operators are utilized and they are randomly picked out each time a new individual is generated:

**one-point row:** A pixels row choosing process is done in random manner., where all the pixels above particular row will come from one of the parent and all the pixels below that particular row will come from the second parent.

**one-point column:** This method process like the above method, but a column is chosen inspite of the row.

**point-to-point random:** This method prefers each pixel from one of the parents in random manner until the new individual is created. In addition mutation process may be performed on each new individual, when the randomly chosen value interval [0, 1] is lower than the mutation rate. Three state-of-the-art image denoising methods are selected arbitrarily and utilized as operators of mutation for each time the mutation rate is satisfied:

- BM3D [3].
- Anisotropic Diffusion [4].
- Wiener-chop [5].

The above mentioned methods are chosen for improved mage denoising efficiency along with reduced computational time during execution. In point of fact, such individual is changed by mutation operators for improving the image restoring efficiency. During evolution ending process, fitness of each individual sorts the intermediate population.

During the next iteration first popSize individuals becomes the population.

- ✓ *Gaussian blur:* This filter does image blur filtering process by utilizing a Gaussian filter, where the size of the filter is selected between 3×3 pixels and 5×5 pixels in random manner.
- ✓ *Averaging filter:* This filter filters noise in the image with an averaging filter whose size is chosen randomly between 3×3 pixels and 5×5 pixels.
- ✓ *Intensity change:* Image pixels value is multiplied by the same factor and the factor is randomly chosen within the interval [0.7, 1.3].

## B. Hybrid filters:

### 1. Adaptive Filter:

Nonlinear filters recognises noise data, locates noise data and remove the same. The algorithm is named 'nonlinear' according as it checks each data point and determines whether the data is noise or valid signal. When noise data is pointed out, an estimate removes and replaces in a simple manner with respect to surrounding data points, and no modification is done on data parts that are not determined as noise. Decision capability is low in the Linear filters, where the linear filters namely band pass, high pass, and low pass, and therefore alter all data. On certain occasions the nonlinear filters are utilized for getting rid of very short wavelength, where the high amplitude is featured from data. Esteemed filter is named as a noise spike-rejection filter, which only performs high short wavelength geological features removing process in an effective manner. Image restoration is a method that involos corrupted/noisy image removing operation and clean original image estimation process. Image Corruption may enter may heaps in many ways such as motion blurs noise.

### 2. Hybrid Median Filter:

Hybrid Median Filter is nonlinear class windowed filter, where the respective filter removes impulse noise and also preserving edges in easy manner. While comparing with Guided Filter basic version, the hybrid filter provides better corner preserving characteristics. The general idea behind filter is continuously applying (image) median technique for any elements of the signal that varys the shape of the window and then removes the median of the got median values. The Hybrid Median Filter induces two medians: in an "X" and in a "+" centered on the pixel. The output of the filter is the median of these two medians and the original pixel value.

$B = \text{hmf}(A, n)$  realizes Hybrid Median Filtering process for matrix  $A$  by utilizing an  $n \times n$  box. The Hybrid Median Filter upholds edges better than a square kernel (neighbour pixels) Guided Filter by performing a three-step ranking operation: The operation includes separately ranking data from differently spatial directions. Three median values are

calculated: MR is horizontal median and vertical R pixels, and MD is D pixels diagonal median. The filtered value is the median of the two median values and the central pixel C: median ([MR, MD, C]).

As an example, for  $n = 5$ :

$$Y = \text{median} \{MR, MD, C\}$$

Hybrid Median Filter algorithm:

1. Place a cross-window over element;
2. Pick up elements;
3. Order elements;
4. Take the middle element;
5. Place a +-window over element;
6. Pick up elements;
7. Order elements;
8. Take the middle element;
9. Pick up result in point 4, 8 and element itself;
10. Order elements;
11. Take the middle element.

All window filters exist with some issues. The issues includes treating the edges. When the window is placed over an element at the edge, window's some part will be empty. For filling the gap formed, signal extension process is performed. Better idea is available for Hybrid Median Filter for extending the images in a symmetrical manner. To put differently lines at the top and at the bottom of the image are added and columns to the left and to the right of it are added respectively. The Hybrid Median Filter leverages in holding the corners and other features that are removed by the  $3 \times 3$  and  $5 \times 5$  Guided Filters. By repeated application, the Hybrid Median Filter fails to excessively smoothen the image details (as do the conventional Guided Filters), and also typically improves superior visual quality in the filtered image. The best part of the Hybrid Median Filter is that it allows the filter to realize better filtering process comparing with the standard Guided Filter on fast-moving picture information of small spatial extent due to adaptive nature of the Hybrid Median Filter.

### 3. Guided Filter

The Guided Filtering process improves edge preserving ability, and avoids introducing new pixel values in the processed image (Wei Fan, et al, 2015). The Guided Filter applies the nonlinear smoothing technique for reducing the edges blurring value; here the main aim is to replace the image current point by the median of the brightness in its neighborhood. The brightness median in the neighborhood is not strucked by individual noise spikes. The impulse noise is filtered by the Guided Filter in an efficient manner. Because of low blurring the edges, this Guided Filter is applied iteratively. The major problem concerned with the Guided Filter is that it is relatively expensive and is hard to compute. The Guided Filter is essential to sort all the values in the

neighborhood into numerical in order to find out the median value which is relatively slow (Vijayalakshmi, et al, 2014).

Guided Filter is based on the following steps: (Er. Amita Kumari, et al, 2014)

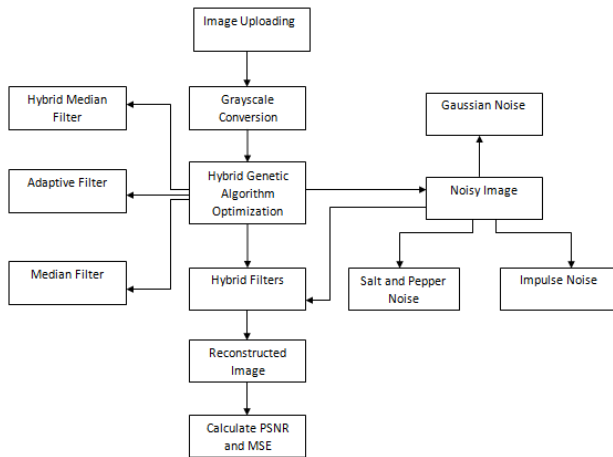
- 1) The filter checks for noisy pixels in the image.
- 2) For each such pixel P, a window of size  $5 \times 5$  around the pixel P is taken.
- 3) Exact differences between the pixel P and the surrounding pixels is found.
- 4) Computes the arithmetic mean (AM) of the differences for a given pixel p.
- 5) The AM is then compared with the —threshold to detect whether the pixel p is informative or corruptive.
  - a) If AM is greater than or equal to the threshold the pixel is considered noisy.
  - b) Otherwise the pixel is considered as information.

The filter fails in performing with higher noise densities. With high noise density it is decided that there might be more informative pixels than corruptive pixels.

## V. EXPERIMENTAL RESULTS

This section is provided with the mixed mode filter results (Hybrid Filter) that is combined result of Adaptive Filter, Hybrid Median Filter and Guided Filter. The experiments are performed on the gray scale images and also on the color images, and the change in PSNR and MSE ratio of the images are observed. The noisy images are assumed by adding Gaussian noise, Salt and Riccian Noise, Impulse noise on the original images. The performance of the method is exemplified with quantitative performance measure. As quantitative measure the peak signal to noise (PSNR) and MSE is used.

## Data Flow Diagram:



The images in Figure 1 were selected to evaluate the Hybrid Genetic Algorithm results. These images were deteriorated with an additive Gaussian noise  $N(0, \sigma)$  with eleven different values for the standard deviation  $\sigma = 10, 15, 20, 25, 30, 35, 40, 45, 50, 55$  and  $60$ .

Hybrid Genetic Algorithm is implemented in Matlab R2015a (8.1.0.604) 64-bit, as well as all other methods used for comparisons. Hybrid Genetic Algorithm is processed with multiple images and for each noise in their image standard deviation level is applied. All the experiments were conducted on an Intel Core i7 920 2.67 GHz processor with 4 GB of RAM.

The following metrics are calculated with the images restored by their respective method. In the case of GA and Hybrid Genetic Algorithm, the image restored is represented by the best individual returned at the end of their execution.

PSNR (peak signal-to-noise ratio): One of the most common metrics, that is represented in decibels (dB) and defined in Equation 2 for 8-bit gray-scale images:

$$PSNR = 10 \log_{10} \left( \frac{255^2}{MSE} \right)$$

The MSE is the mean squared error formed between the original and the recovered images. It is defined in Equation 3, where  $M$  and  $N$  are the dimensions of the image.

$$MSE = \frac{1}{MN} \sum_{i=0}^{M-1} \sum_{j=0}^{N-1} [O(i, j) - K(i, j)]^2$$

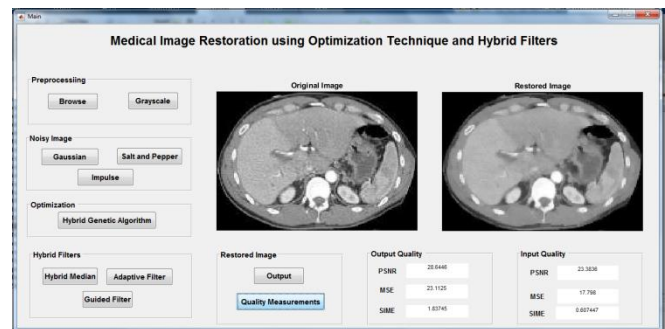
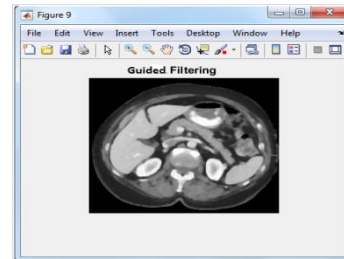
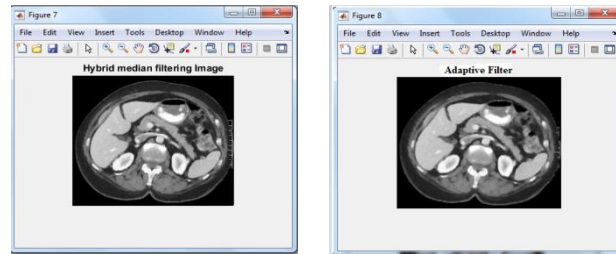
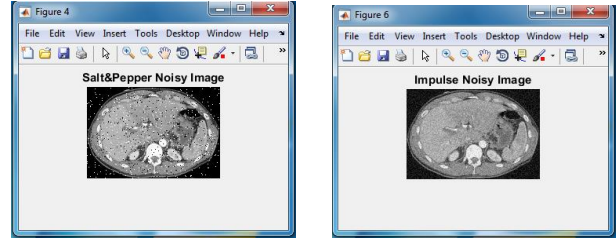
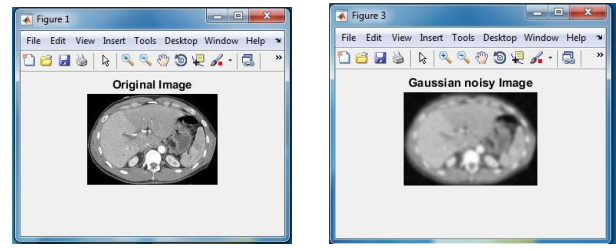


Figure Shows the images corrupted with different noise variance are filtered with mixed mode filtering and describe the PSNR and MSE

## VI. CONCLUSION

This paper that is discussed above applies Hybrid Genetic Algorithm for performing image denoising process along with optimization technique, where three different image

denoising techniques were used as mutation operators and allowed to initialize and reinitialize the populations. Multiple levels of noise were added to images and where was evaluated against other state-of-the-art approaches and the results are compared with Hybrid Genetic Algorithm results. The Hybrid Genetic Algorithm outperformed a previous GA applied to the image denoising problem, which points that the GA combined with specific methods for image denoising can bring significant gain. The paper concludes that Hybrid Genetic Algorithm provides competitive results compared with other techniques available in the literature, especially in images with high noise level.

Hybrid Genetic Algorithm is so far time consuming when compared to certain image denoising techniques. As future work, we aim to parallelize the execution of the mutation operators and also to investigate new fitness functions in such a way oftenly best individuals are found. Furthermore, proposing different crossover operators to segment the images and pass specific segments to the next generation can be along with other image denoising techniques as mutation operators.

#### ACKNOWLEDGMENT

This work was done on fulfillment of my final year project work on guidance of staff members of Department of Computer Science, VIT University, Chennai. Thanks to Mr. XXXX For valued guidance to complete this Journal.

#### REFERENCES

- [1] J. Amudha, N. Pradeepa, and R. Sudhakar, "A Survey on Digital Image Restoration," *Procedia Engineering*, vol. 38, pp. 2378–2382, 2012, international Conference on Modelling Optimization nad Computing.
- [2] J. Mohan, V. Krishnaveni, and Y. Guo, "A Survey on the Magnetic Resonance Image Denoising Methods," *Biomedical Signal Processing and Control*, vol. 9, pp. 56–69, 2014.
- [3] K. Dabov, A. Foi, V. Katkovnik, and K. Egiazarian, "Image Denoising with Block-Matching and 3D Filtering," in *SPIE Electronic Imaging: Algorithms and Systems*, vol. 6064, Jan. 2006, pp. 606 414–1–606 414– 12.
- [4] M. J. Black, G. Sapiro, D. H. Marimont, and D. Heeger, "Robust Anisotropic Diffusion," *IEEE Transactions on Image Processing*, vol. 7, no. 3, pp. 421–432, Mar. 1998.
- [5] S. Ghael, E. P. Ghael, A. M. Sayeed, and R. G. Baraniuk, "Improved Wavelet Denoising via Empirical Wiener Filtering," in *Proceedings of SPIE*, vol. 3169, San Diego, CA, USA, Jul. 1997, pp. 389–399.
- [6] R. C. Gonzalez and R. E. Woods, *Digital Image Processing (3rd Edition)*. Upper Saddle River, NJ, USA: Prentice-Hall, Inc., 2006.
- [7] F. Russo, "A Method for Estimation and Filtering of Gaussian Noise in Images," *IEEE Transactions on Instrumentation and Measurement*, vol. 52, no. 4, pp. 1148–1154, Aug. 2003.
- [8] —, "Image Filtering based on Piecewise Linear Models," in *IEEE International Workshop IST*, Stresa, Italy, May 2004, pp. 7–12.
- [9] L. Rudin, S. Osher, and E. Fatemi, "Nonlinear Total Variation based Noise Removal Algorithms," *Physica D*, vol. 60, pp. 259–268, 1992.
- [10] A. Chambolle, "An Algorithm for Total Variation Minimization and Applications," *Journal of Mathematical Imaging and Vision*, vol. 20, no. 1-2, pp. 89–97, 2004.
- [11] C. Drapaca, "A Nonlinear Total Variation-based Denoising Method with two Regularization Parameters," *IEEE Transactions on Biomedical Engineering*, vol. 56, no. 3, pp. 582–586, 2009.
- [12] P. Perona and J. Malik, "Scale-Space and Edge Detection using Anisotropic Diffusion," *IEEE Transactions on Pattern Analysis and Machine Intelligence*, vol. 12, no. 7, pp. 629–639, 1990.
- [13] V. Katkovnik, K. Egiazarian, and J. Astola, *Local Approximation Techniques in Signal and Image Processing*. SPIE Press, Sep. 2006, vol. PM157.
- [14] R. D. da Silva, R. Minetto, W. R. Schwartz, and H. Pedrini, "Adaptive Edge-Preserving Image Denoising using Wavelet Transforms," *Pattern Analysis and Applications*, vol. 16, no. 4, pp. 567–580, 2013.
- [15] L. Sendur and I. W. Selesnick, "Bivariate Shrinkage with Local Variance Estimation," *IEEE Signal Processing Letters*, vol. 9, no. 12, 2002.
- [16] J. Orchard, M. Ebrahimi, and A. Wong, "Efficient Nonlocal-Means Denoising using the SVD," in *IEEE International Conference on Image Processing*, San Diego, CA, USA, Oct. 2008, pp. 1732–1735.
- [17] Y. Wongsawat, K. Rao, and S. Orintara, "Multichannel SVD-based Image De-noising," in *IEEE International Symposium on Circuits and Systems*, vol. 6, May 2005, pp. 5990–5993.
- [18] J. A. Sethian, *Level Set Methods and Fast Marching Methods: Evolving Interfaces in Computational Geometry, Fluid Mechanics, Computer Vision and Materials Sciences*. Cambridge University Press, 1999.
- [19] G. Fan and X. Xia, "Image Denoising using Local Contextual Hidden Markov Model in the Wavelet Domain," *IEEE Signal Processing Letters*, vol. 8, no. 5, pp. 125–128, May 2001.
- [20] J. K. Romberg, H. Choi, and R. G. Baraniuk, "Shift-Invariant Denoising using Wavelet-domain Hidden Markov Trees," in *Conference Record of The Thirty-Third Asilomar Conference on Signals, Systems and Computers*, Pacific Grove, CA, USA, Oct. 1999, pp. 1277–1281.
- [21] V. Gupta, C. C. Chan, and P. T. Sian, "A Differential Evolution Approach to PET Image De-noising," in *Engineering in Medicine and Biology Society, 2007. EMBS 2007. 29th Annual International Conference of the IEEE*, Aug 2007, pp. 4173–4176.
- [22] V. Thavavel, J. J. Basha, M. Krishna, and R. Murugesan, "Heuristic Wavelet Approach for Low-Dose EPR Tomographic Reconstruction: An Applicability Analysis with Phantom and in Vivo Imaging," *Expert Systems with Applications*, vol. 39, no. 5, pp. 5717–5726, 2012.
- [23] C. Toledo, L. de Oliveira, R. Dutra da Silva, and H. Pedrini, "Image Denoising based on Genetic Algorithm," in *IEEE Congress on Evolutionary Computation (CEC)*, Jun. 2013, pp. 1294–1301.
- [24] H. Ishikawa, "Global Optimization Using Embedded Graphs," Ph.D. dissertation, Department of Computer Science, New York University, 2000.
- [25] D. Zosso and A. Bustina, "A Primal-Dual Projected Gradient Algorithm for Efficient Beltrami Regularization," 2014, preprint.